TACIT MATHS

- Indices
- Logs

Name:_____

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Recap of Indices

Recap

Rules of Indices

(Log tables pg21)

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{a^p}$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Indices Ex. 1a

Express in the form 5^n

25	125	$(125)^4$	$\frac{1}{25}$	$\sqrt{5}$	$\sqrt{125}$

Express in the form 2^n

8	$\frac{1}{8}$	64^2	$\sqrt{32}$	$\frac{1}{\sqrt{32}}$	$\frac{8}{\sqrt{32}}$

Indices Ex. 1c

Express the following in the form: x^n

$$\frac{1}{x^2} \qquad \frac{3}{x} \qquad \sqrt{x} \qquad \sqrt{x^5} \qquad \frac{1}{\sqrt{x}} \qquad \frac{3}{\sqrt{x^3}}$$

Indices Ex. 1d

Express in the form $(3x-1)^n$

$$\sqrt{3x-1} \qquad \frac{1}{(3x-1)^3} \qquad \frac{1}{3x-1} \qquad \frac{1}{\sqrt{3x-1}}$$

Ex. 2: Indices Equations

Solve

$$2^x = \frac{1}{16}$$

Solve

$$3^x = 3\sqrt{3}$$

Solve

$$3^x = \frac{9}{\sqrt{27}}$$

Solve

$$\frac{1}{25^x} = \frac{125}{\sqrt{5}}$$

Logs

$$a^x = b$$
$$\log_a b = x$$

Ex. 3: Logs Intro

Solve
$2^x = 64$

Solve
$2^x = 50$

Solve
$2^x = 300$

Rules of Logs

(Log tables pg21)

$\log_a xy = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^n = n\log_a x$

$\log_a a = 1$

$\log_a 1 = 0$

$\log_a x = \frac{\log_b x}{\log_b a}$

Ex.4a: Logs

Solve
$\log_5 125 = x$

Solve
$\log_x 81 = 4$

Solve
$\log_5 x = 4$

Solve
$\log_2 \sqrt{8} = x$

Solve
$\log_{36} x = \frac{1}{2}$

Solve
$\log_3 1 = x$

Ex. 4b

Simpify	*Simplify*	*Simplify*
$\log_4 32 + \log_4 2$	$3\log_8 4$	$\log_3 2 + 2\log_3 3 - \log_3 18$

Ex. 5: "Changing the Base"

Simplify the following, to 4 decimal places using the "Common Log" (i.e. $\log_{10}$*)*

$\log_4 7$	$\log_9 16$	$\log_{100} 10000$

Show that

$$\log_{25} x = \frac{1}{2}\log_5 x\,.$$

Ex. 6: Logarithmic Equations

Solve

$$\log_2(x-2)+\log_2 x=3$$

Solve

$$\log_3(x^2+8x)-\log_3(x^2-2x)=1$$

Ex. 7: Harder *Looking* Questions

Use your calculator and the rules of Logs to solve the following:

$$(1.1098)^t=2.18441$$

$$3.2(5.55)^c=7.5387$$

$$1.2-6.1(4.22)^m=1.195442$$

(b) **(i)** Find the value of x, where $x \in \mathbb{N}$ given

$$\log_2(x+2) + \log_2(x-2) = 5.$$

(ii) Solve for x. $2^x + 2^{1-x} = 3.$

Ex. 9: ExamCraft 2017

(b) **(i)** Solve the equation:

$$x + \frac{1}{x} = \frac{10}{3}, \quad \text{where } x \in \mathbb{R} \setminus \{0\}.$$

(ii) Use your answers above to solve the equation:

$$9^x + \frac{1}{9^x} = \frac{10}{3}, \quad \text{where } x \in \mathbb{R}.$$

(iii) Hence, or otherwise, solve the equation:

$$\log_3 x + \log_x 3 = \frac{10}{3}, \quad \text{where } x > 1.$$

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Ex. 10: DEB 2017

(b) **(i)** Express $\log_9 xy$ in terms of $\log_3 x$ and $\log_3 y$.

(ii) Hence, or otherwise, solve the simultaneous equations for x and y:

$$\log_9 xy = \frac{5}{2}$$
$$\log_3 x . \log_3 y = -6.$$

Express your answers in their simplest form.

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Ex. 11: DEB 2018

Question 3 **(25 marks)**

(a) Solve the equation $3^{2x+2} - 28(3^x) + 3 = 0$. [Hint: Let $y = 3^x$.]

Ex. 12: Leaving Cert 2016 Q4(b)

(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

(ii) $\log_a \frac{9a^2}{16}$.

Ex. 13: DEB 2019

(b) Solve the simultaneous equations:

$$\log(x+y) = 2\log x$$
$$\log y = \log 2 + \log(x-1), \text{ where } x > 1, y > 0.$$

(b) The heptathlon is an Olympic competition. It consists of seven events including the 200 m race and the javelin. The scoring system uses formulas to calculate a score for each event. The table below shows the formulas for two of the events and the values of constants used in these formulas, where x is the time taken (in seconds) or distance achieved (in metres) by the competitor and y is the number of points scored in the event.

Event	x	**Formula**	a	b	c
200 m race	Time (s)	$y = a(b - x)^c$	4·99087	42·5	1·81
Javelin	Distance (m)	$y = a(x - b)^c$	15·9803	3·8	1·04

(i) In the heptathlon, Jessica ran 200 m in 23·8 s and threw the javelin 58·2 m. Use the formulas in the table to find the number of points she scored in each of these events, correct to the nearest point.

200 m

Javelin

(ii) The world record distance for the javelin, in the heptathlon, would merit a score of 1295 points. Find the world record distance for the javelin, in the heptathlon, correct to two decimal places.

(iii) The formula used to calculate the points for the 800 m race, in the heptathlon, is the same formula used for the 200 m race but with different constants.
Jessica ran the 800 m race in 2 minutes and 1·84 seconds which merited 1087 points.
If $a = 0{\cdot}11193$ and $b = 254$ for the 800 m race, find the value of c for this event, correct to two decimal places.

Ex. 15: 2014 Examcraft

Question 8 **(35 marks)**

In 1935, while working at the California Institute of Technology, Charles Francis Richter invented the Richter scale. The Richter scale is used to measure earthquake intensity on a machine called a seismograph.

The formula for the Richter is given by $M = \log_{10}\left[\frac{I}{I_0}\right]$

where M is the magnitude on the Richter scale in millimetres, I_0 is the 'threshold quake intensity', i.e. a movement that can barely be detected, and I is the intensity of the earthquake in millimetres. $I_0 = 10^{-3}$ mm.

(a) Calculate the magnitude of an earthquake with an intensity of 500 mm.

(b) In 2005 an earthquake in Kashmir in Pakistan measured 7.5 on the Richter scale. Calculate the intensity of the earthquake.