THE Exponential

Factor Theorem - LCHL

Name: _____

Table of Contents

NOTES:

Other Equations on the LC Course

Types of Equations:

Quadratic Equations
$$x^2 + 13x - 30 = 0$$

Linear Equations
$$3(4 - 3x) = 5(3 - 2x)$$

Modular Equations/Inequalities
$$|1 + 2x| < |x + 2|$$

Algebra Fractions:
$$\frac{1}{x - 1} + \frac{4}{x} = 3$$

Cubic Equations
$$x^3 + 2x^2 - 11x - 12 = 0$$

Inequalities
$$\frac{2x - 7}{x + 3} > 1$$

Surds
$$\sqrt{2x + 7} - 2 = x$$

Ex. 1a :Solving Exponential Equations

(a)
$$e^t = 1096.63316$$

(b)
$$e^{0.234b} = 13.11819$$

(c)
$$3.456e^{-0.09m} = 2.20364$$

(d)
$$ln7x^2 = 4.451436$$

NOTES:

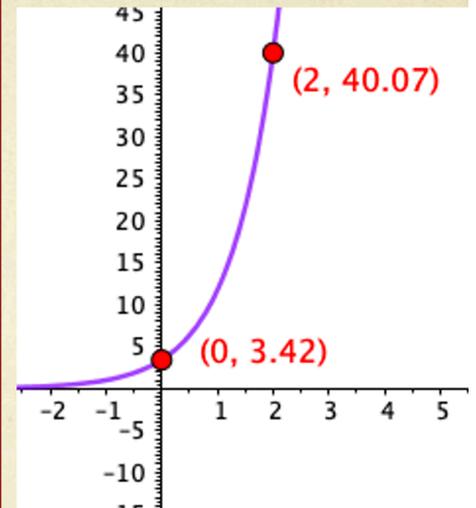## Ex. 1b: THE Exponential Equations V Exponential Equations

$e^t = 22$	$5^x = 125$	$7^x = 100$

Ex. 1c: Sample Exercise:

Find the value of "a" and "b" in this function.

$$f(x) = ae^{bx}$$

(2, 40.07)

(0, 3.42)

NOTES:

Exam Questions

Ex. 2: DEB 2017

Question 5   **(25 marks)**

(a) The power supply to a space satellite is provided by means of a generator that converts heat released by the decay of a radioisotope into electricity. The power output, in watts, may be calculated using the function

$$w(t) = A e^{bt},$$

where t is the time, in days, from when the satellite is launched into space. The initial power output at the launch of the satellite is 60 watts.

(i) Given that after 14 days the power output falls to 56 watts, calculate the value of b, correct to three decimal places.

(ii) The satellite cannot function properly when the power output falls below 5 watts. After how many days will the satellite fail to function properly?

(b) Find the value of the constant k for which $w(t+k) = \dfrac{1}{2}w(t)$, for all $t \geq 0$.

Give your answer in the form $p\ln q$, where $p, q \in \mathbb{N}$.
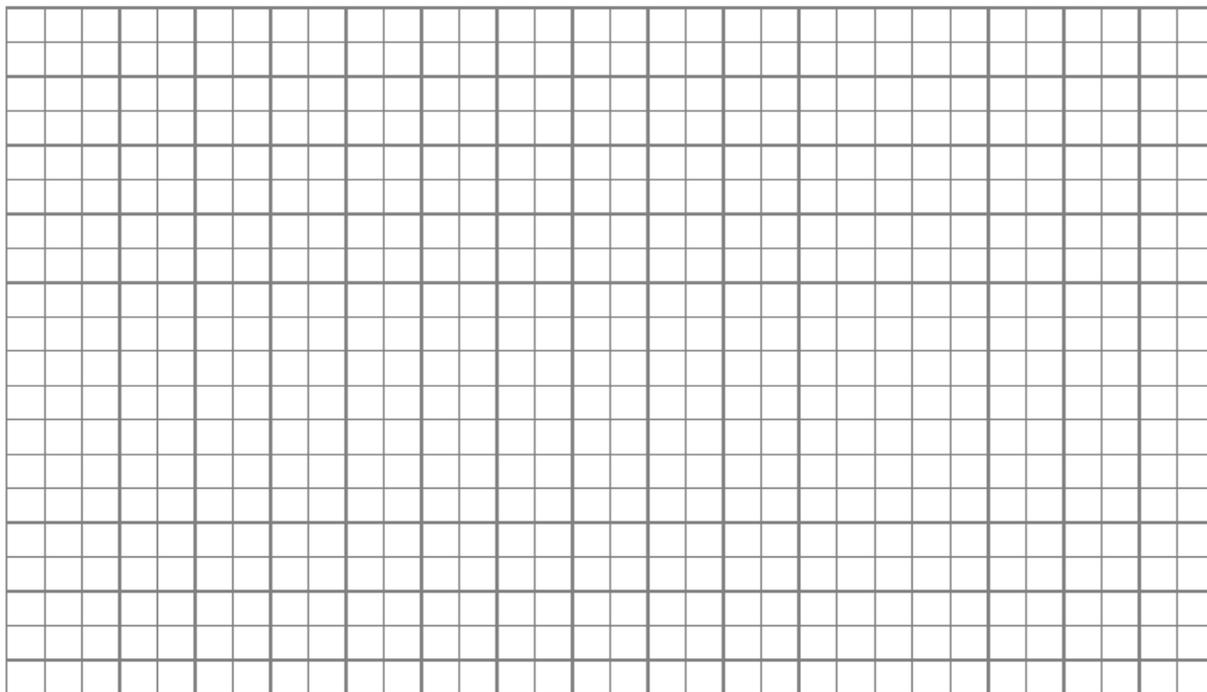
| Previous | Page | Running |

NOTES:

Ex. 3: Examcraft 2014

(b) The growth of a certain species of bacteria can be modelled with the equation $N = Re^{kt}$.
A science teacher puts 100 bacteria into nutrient agar plates (agar will act as a food source for the bacteria). Five hours later, there are 600 bacteria.

(i) Calculate the value of k correct to four decimal places.

(ii) After how many hours will the number of bacteria in the plate be 12,000?

NOTES:

Ex. 4: 2016 ExamCraft
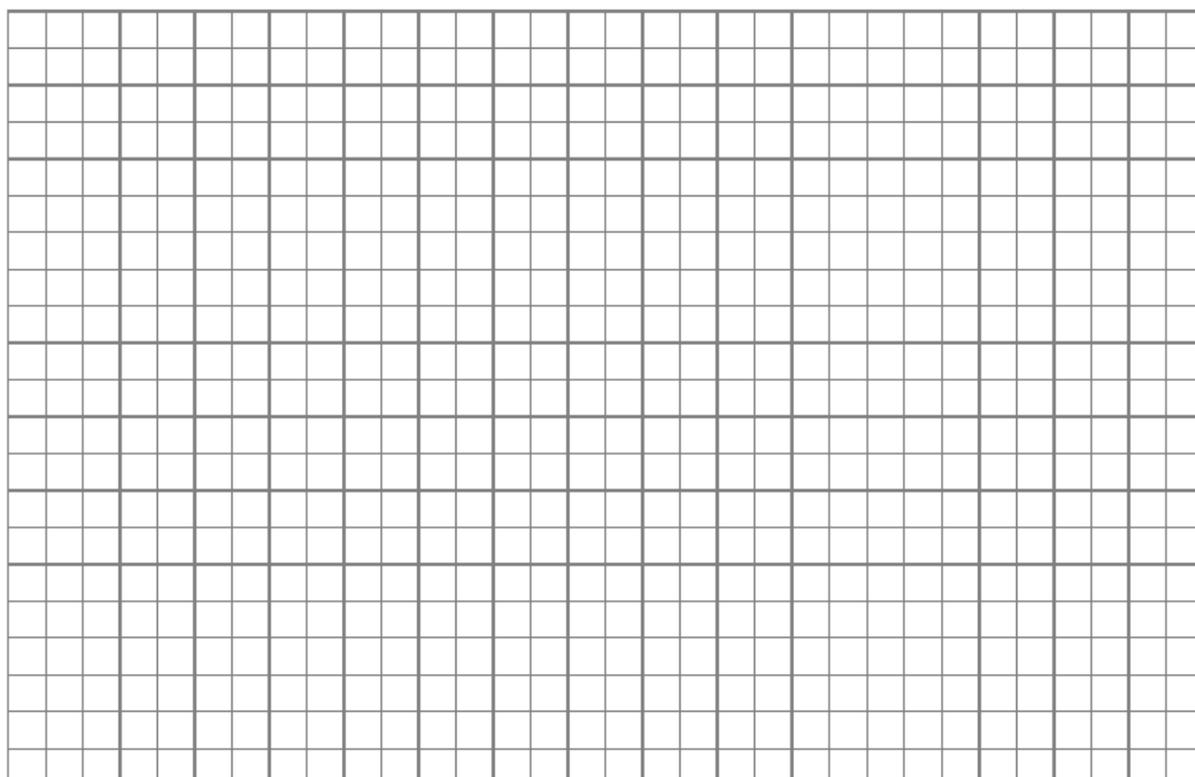
**Question 9**   **(40 marks)**

Carbon-14 is a radio-isotope of the element Carbon that is used in carbon dating. Its decay can be modelled by the function

$$F = Be^{kt}$$

where F is the final mass remaining, in kg, at any time t, in years, and B is the initial amount, in grams, of Carbon-14 and k is the decay constant. The half-life (the time it takes for half of the mass to decay) of Carbon-14 is 5730 years.

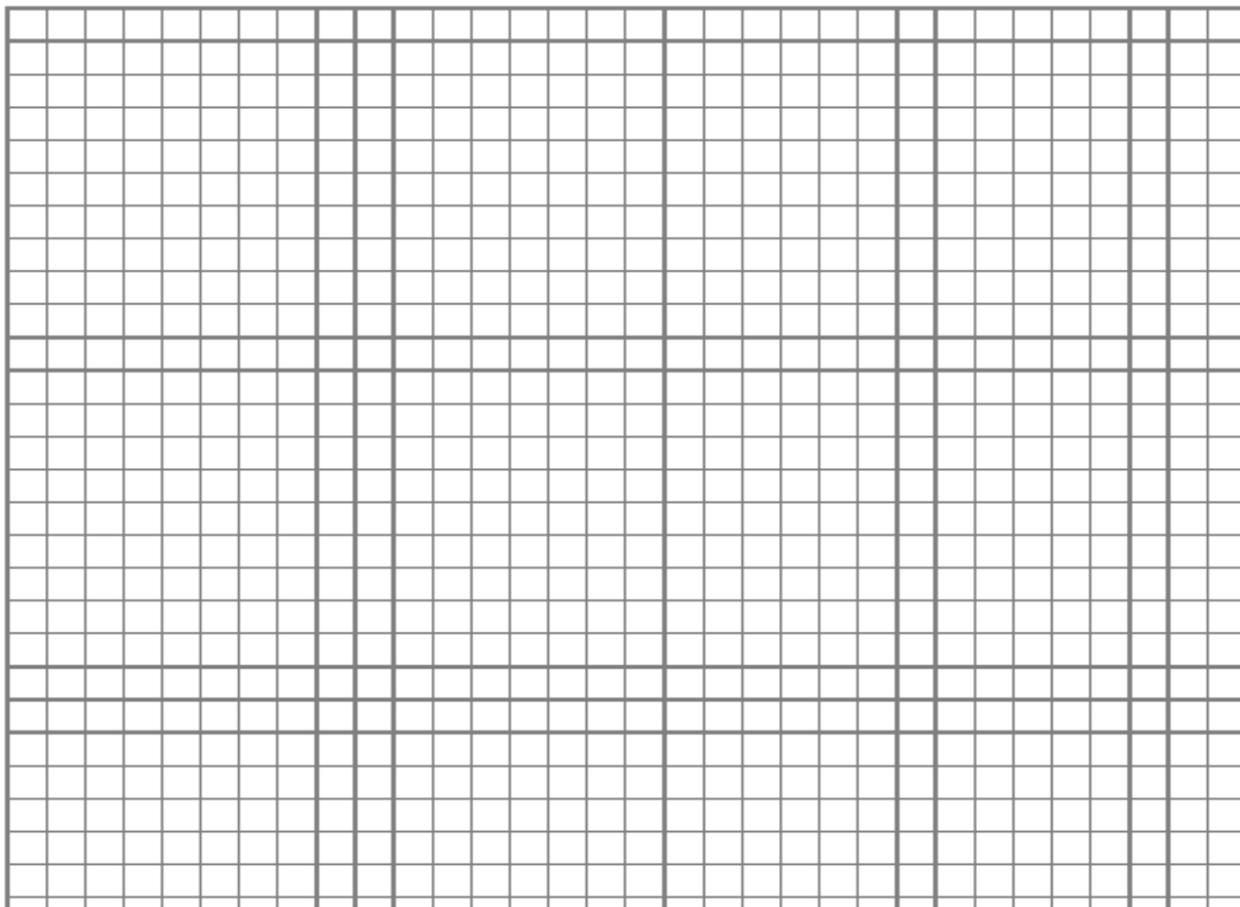**(a)** If the original mass of Carbon-14 is 1kg, show that $k = \dfrac{\ln(0.5)}{5730}$.
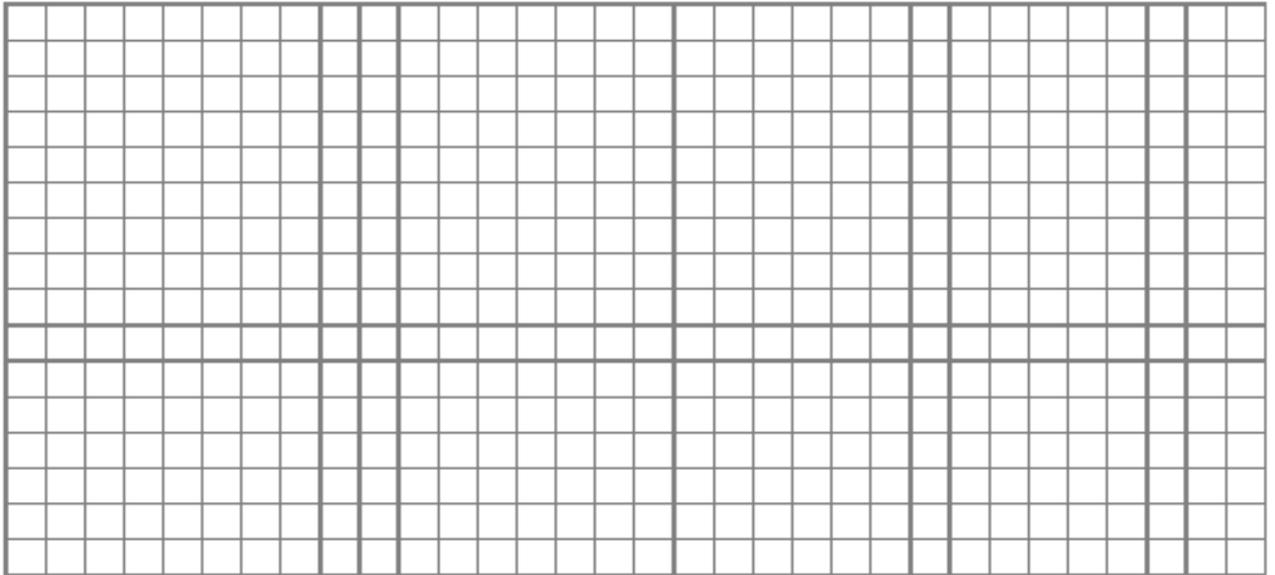
A research scientist is presented with a document which contains the writings of a soldier from the Trojan War circa 1250 BC (around 3250 years ago). After testing it is found that the parchment contained $1 \times 10^{-12} g$ of Carbon-14.

**(b)** Calculate the original amount of Carbon-14 in the parchment.
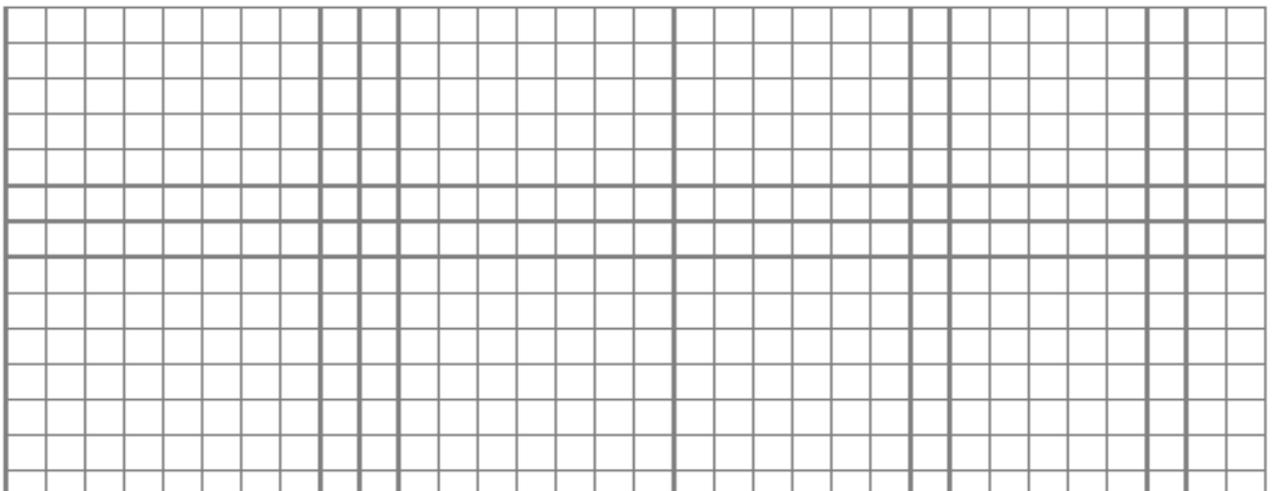
NOTES:

(c)   It is known that parchments from this area usually contain in the region of $1.3 \times 10^{-12} g$ of Carbon-14. Taking the final amount as $1 \times 10^{-12} g$ calculate the age of the document and explain if you believe it to be genuine or a fake.

NOTES:

Ex.5: Examcraft 2019

Question 7   **(65 marks)**

The rate at which an object's temperature changes is approximately proportional to the difference between the object's temperature and the temperature of its surroundings. This is called Newton's law of cooling and is written as,

$$T - T_s = (T_0 - T_s)e^{-kt}.$$

Where T is the temperature of the object at any time t in minutes, T_s is the temperature of the surroundings and T_0 is the initial temperature of the object.

In an office, coffee is brewed in a coffee machine. The office is air-conditioned and held at a constant temperature of 21°C. The coffee machine beeps when it reaches a temperature of 98°C to indicate the coffee is ready.

An employee takes a cup of coffee from the machine and leaves it on a desk.

(a)   Show, mathematically, that $T_0 = 98$°C.

(b)   After 5 minutes the temperature of the coffee is 63°C.
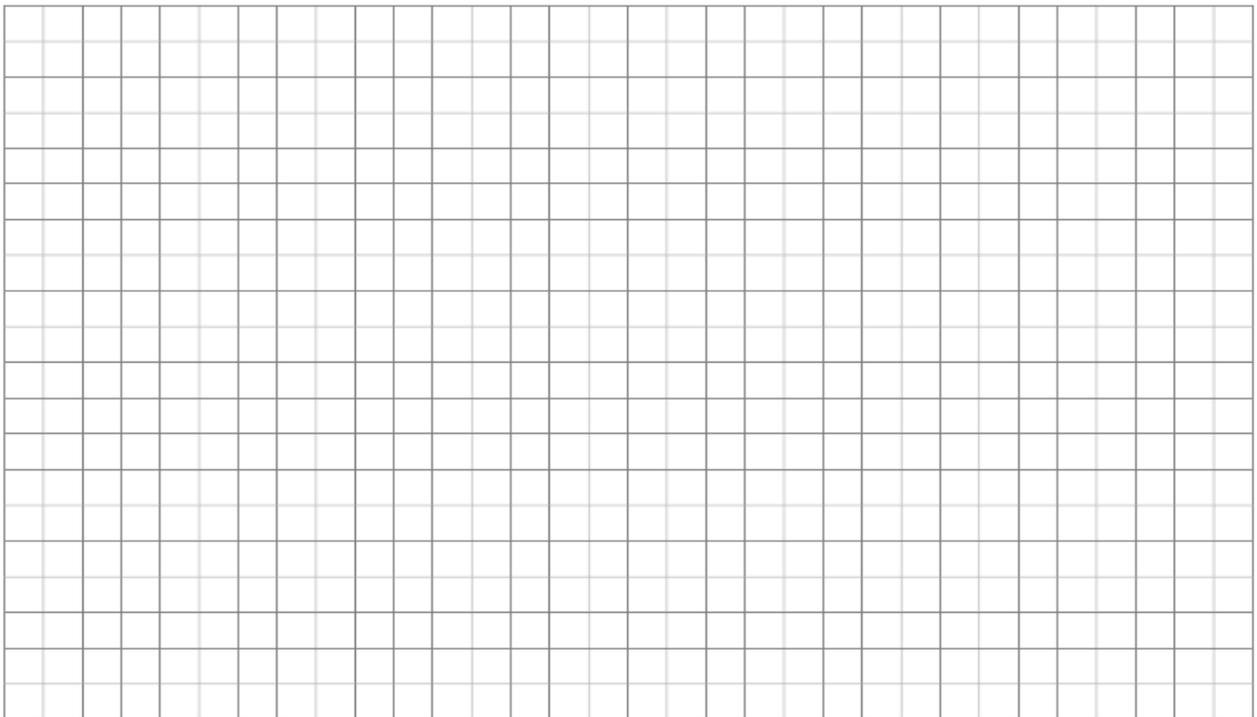   Verify that the value of $k = 0 \cdot 12123$.

NOTES:

(c)   Find the temperature of the cup of coffee after 9 minutes, to the nearest degree.
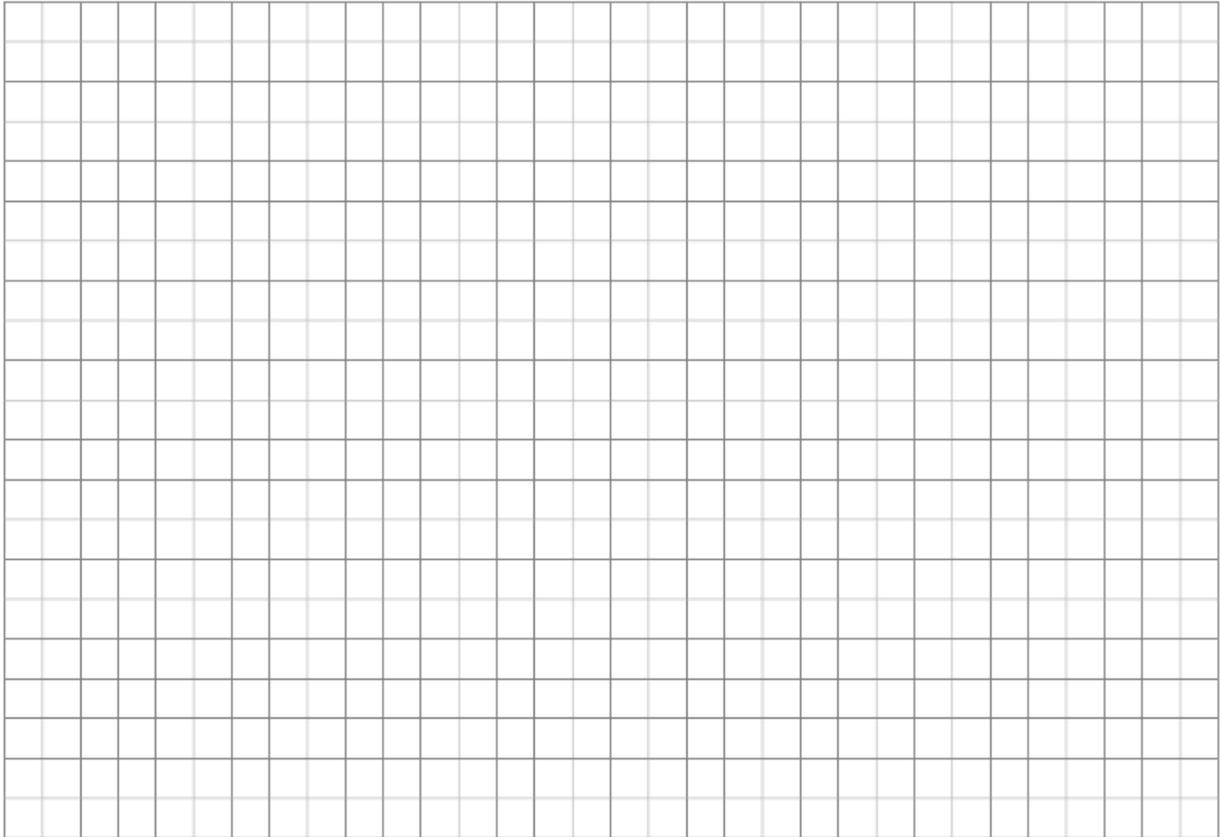
(d)   Coffee experts state that coffee loses its taste quality at a temperature of 22°C. After how many minutes will the cup of coffee lose its taste quality? Give your answer correct to two decimal places.
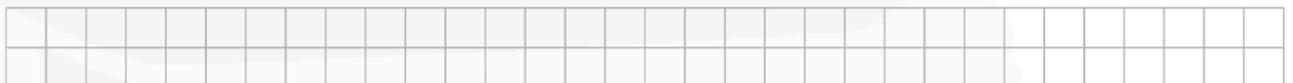
NOTES:

(e) During cold months the air-conditioner has its temperature raised by 5°C. The coffee machine runs at the same temperature all year round. In your opinion, how will the change in temperature affect the cooling rate of a cup of coffee? Verify your answer by comparing the time taken to cool to 63°C.

Ex. 6: Leaving Cert 2018

Question 7   **(55 marks)**

The time, in days of practice, it takes Jack to learn to type x words per minute (wpm) can be modelled by the function:

$$t(x) = k\left[\ln\left(1 - \frac{x}{80}\right)\right], \text{ where } 0 \leq x \leq 70, x \in \mathbb{R}, \text{ and } k \text{ is a constant.}$$

(a) Based on the function $t(x)$, Jack can learn to type 35 wpm in 35·96 days. Write the function above in terms of k and **hence** show that $k = -62·5$, correct to 1 decimal place.

(b) Find the number of wpm that Jack can learn to type with 100 days of practice. Give your answer correct to the nearest whole number.

NOTES:

(c)   Complete the table below, correct to the nearest whole number **and hence** draw the graph of $t(x)$ for $0 \leq x \leq 70, \ x \in \mathbb{R}$.

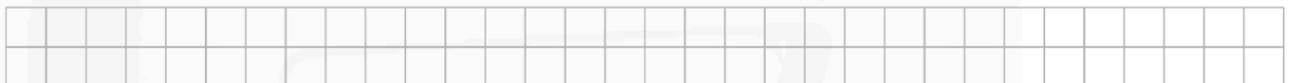| x (wpm)	0	10	20	30	40	50	60	70
$t(x)$ (days)								
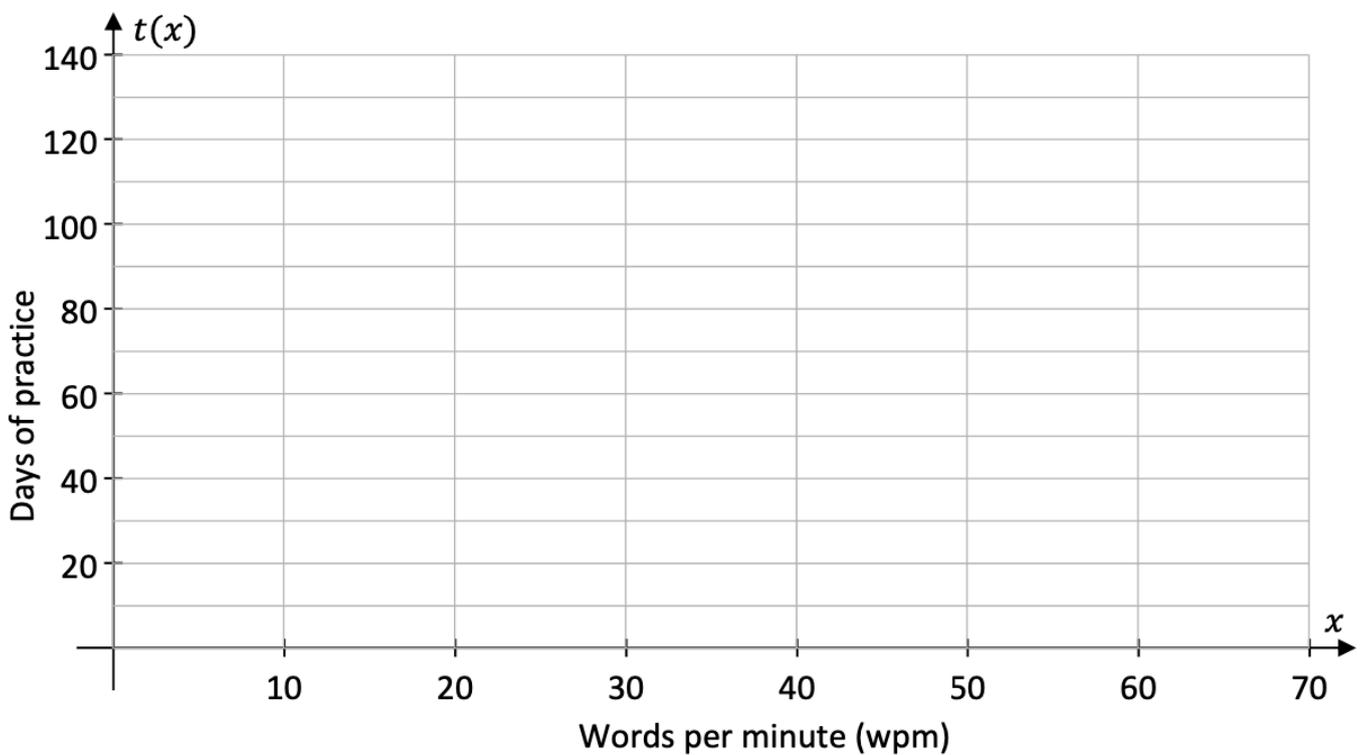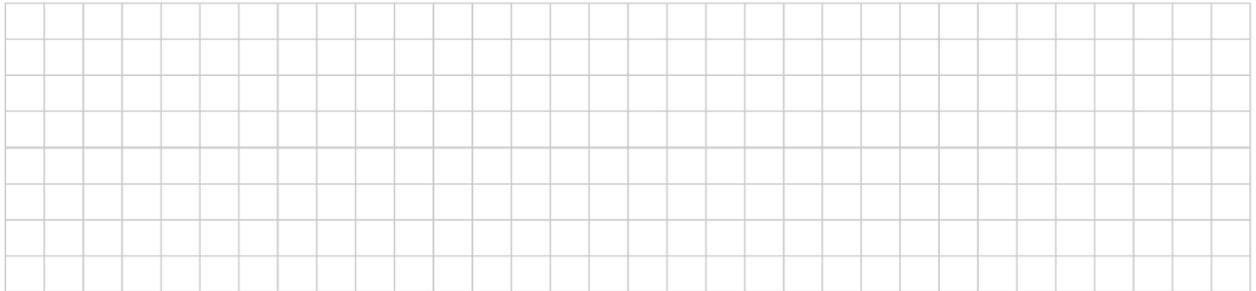
(d)   A simpler function that could also be used to model the number of days needed to attain x wpm is $p(x) = 1{\cdot}5x$.

Draw, on the diagram above, the graph of $p(x)$ for $0 \leq x \leq 70, \ x \in \mathbb{R}$.

(e)   Let $h(x) = p(x) - t(x)$.

   (i)   Use your graphs above to estimate the solution to $h(x) = 0$ for $x > 0$.

NOTES:

Ex. 7: Leaving Cert 2017

Question 7   **(55 marks)**

Sometimes it is possible to predict the future population in a city using a function.
The population in Sapphire City, over time, can be predicted using the following function:

$$p(t) = Se^{0.1t} \times 10^6.$$

The population in Avalon, over time, can be predicted using the following function:

$$q(t) = 3.9e^{kt} \times 10^6.$$

In the functions above, t is time, in years; $t = 0$ is the beginning of 2010; and both S and k are constants.

(a)   The population in Sapphire City at the beginning of 2010 is 1 100 000 people.
Find the value of S.

(b)   Find the predicted population in Sapphire City at the beginning of 2015.

**(c)**   Find the predicted change in the population in Sapphire City during 2015.

NOTES:

(d)   The predicted population in Avalon at the beginning of 2011 is 3 709 795 people.
Write down and solve an equation in k to show that $k = -0.05$, correct to 2 decimal places.

(e)   Find the year during which the populations in both cities will be equal.

(f)   Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

NOTES:

(g) Use the function $q(t) = 3 \cdot 9e^{-0.05t} \times 10^6$ to find the predicted rate of change of the population in Avalon at the beginning of 2018.

NOTES:

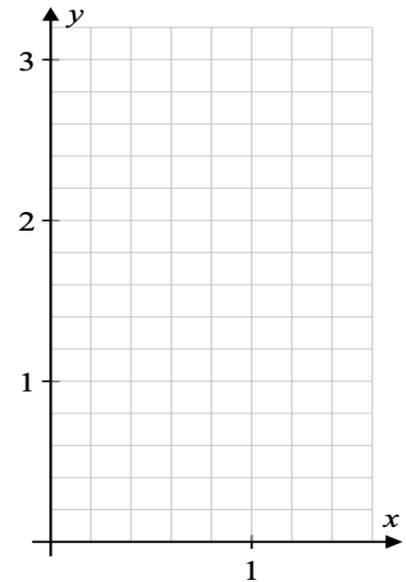## Ex. 8: LC 2016
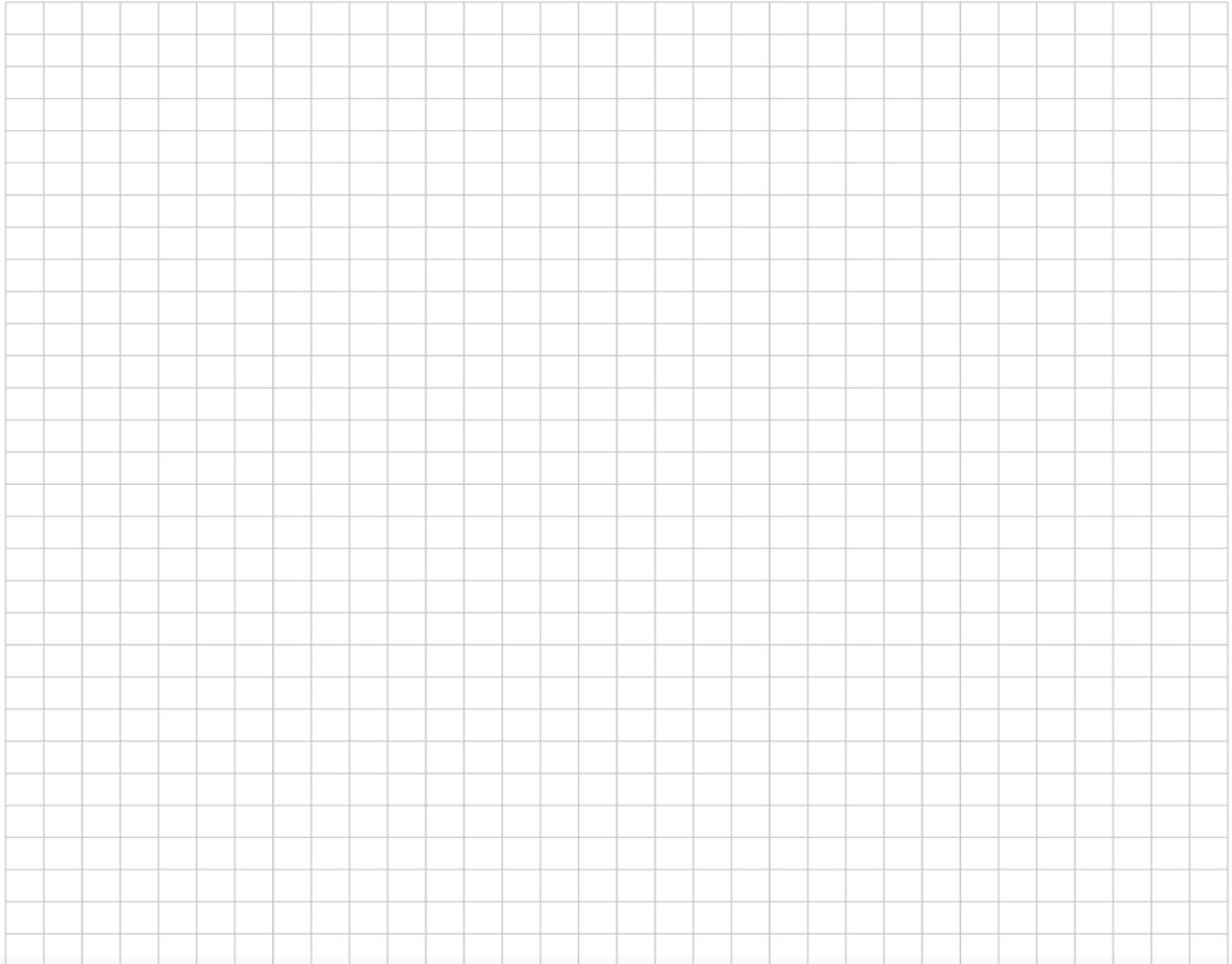
Question 3   **(25 marks)**

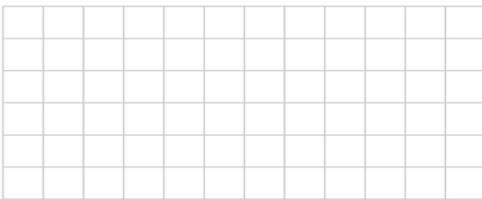**(a)**   **(i)** $f(x) = \dfrac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.

Complete the table below. Write your values correct to two decimal places where necessary.

x	0	$0{\cdot}5$	1	$\ln(4)$
$f(x) = \dfrac{2}{e^x}$				
$g(x) = e^x - 1$				

(ii)   In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \le x \le \ln(4)$. Label each graph clearly.

(iii)   Use your graphs to estimate the value of x for which $f(x) = g(x)$.

(b)   Solve $f(x) = g(x)$ using algebra.

NOTES: